Single-Machine Scheduling with Release Times, Deadlines, Setup Times, and Rejection

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Abstract based on a paper in European Journal of Operational Research [3].

1 Problem description

Single-machine scheduling where jobs have a penalty for being late or for being rejected altogether is an important (sub)problem in manufacturing, logistics, and satellite scheduling. In the 3-field notation [5] this is:

$$1 \mid r_j; s_{jk}; \text{reject}; \bar{d}_j \mid \sum_{j \notin R} w_j T_j - \sum_{j \notin R} v_j,$$

where tardiness $T_j = \max\{f_j - d_j, 0\}$, and R is the set of rejected jobs. The equivalent positive representation of this problem is the order *acceptance* and scheduling (OAS) problem, which aims to maximize the revenue of accepted jobs $\sum_{j \notin R} v_j$ minus their tardiness penalty $\sum_{j \notin R} w_j T_j$.

For an example instance of OAS, please see Figure 1. Note that the number of overlapping time intervals in this instance is limited. This is what we call the *width* of the instance. This width is significantly smaller than the total number of jobs in most real-life situations, because the length of time-windows is usually significantly smaller than the problem horizon. For example in satellite scheduling, jobs are only available when the satellite is near the location in its orbit [4], or in vehicle routing time slots for delivery may be given in minutes while the schedule is in hours.

2 Approach

Our main contribution is to exploit the fact that in many instances this width is limited; secondly we use state dominance in a dynamic programming formulation for this problem. This is similar to the concept of state merging in recent literature on optimization using decision diagrams [1].

These ideas allow us to show that OAS is fixed-parameter tractable (FPT) in the combination of the width w *and* the slack σ (i.e., $\max_j\{\bar{d}_j - p_j - r_j\}$). The FPT algorithm has a runtime bound of $O(n^2 \cdot w^2 \sigma 2^w)$. Second, using only w as a parameter, a fully polynomial-time approximation scheme FPTAS is presented with a runtime bound of $O(\frac{n^3 \cdot w^2 2^w}{\epsilon^2})$. For instances with large width, we provide a Balas-neighbourhood heuristic enforcing such a limited width artificially.

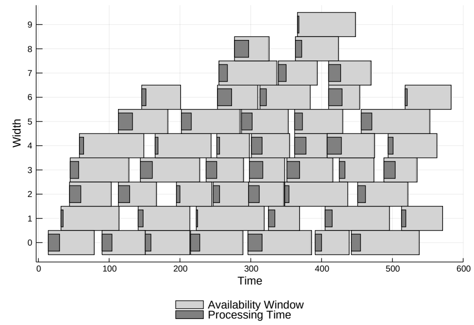

Fig. 1. Example problem instance of OAS for $n = 50$, tardiness $\tau = 0.9$, and $R = 0.1$ as generated according to standard OAS benchmark set.[2]

3 Results

Simulation experiments show that for almost all instances the FPT algorithm is faster than the state-of-the-art branch-and-price (B&P) method [6], sometimes significantly so (e.g., for $n = 100$ optimal solutions were found for some instances in less than 3 minutes where B&P hits the time limit of one hour). This algorithm finds optimal solutions for instances that had not been solved to optimality before, which contributes to our understanding of the quality of heuristics.

When benchmarked against recently published state-of-the-art heuristics, the FPTAS approximation algorithm is only competitive for a smaller width (11 or less), but the Balas heuristic outperforms state-of-the-art heuristics under a wide range of conditions, depending on the choice for the parameter.

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